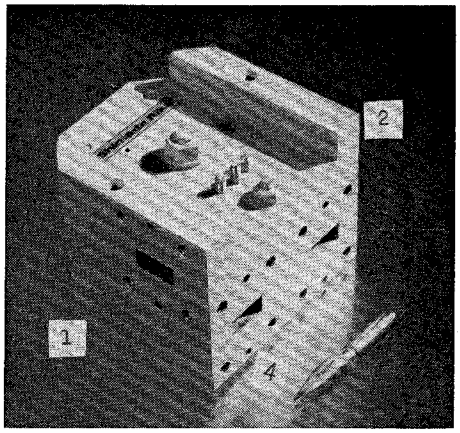
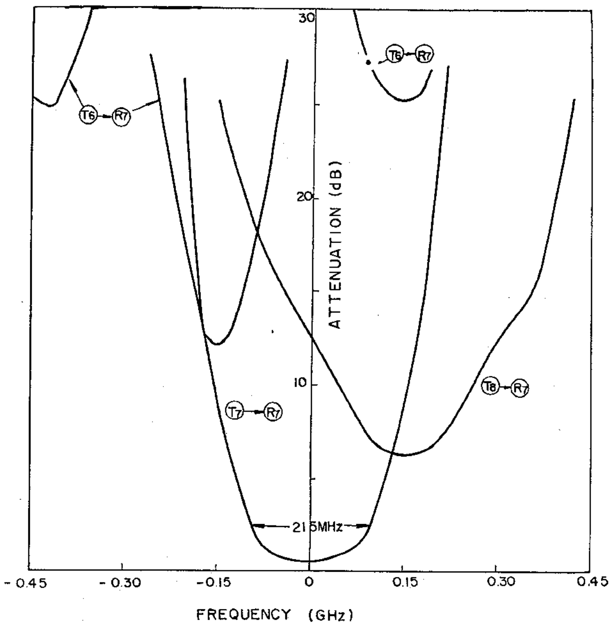
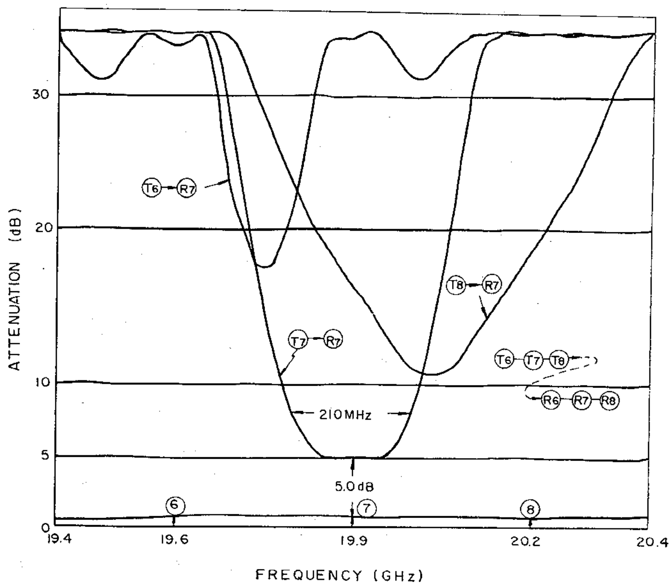


Fig. 4. Trial ring-type channel-dropping filter.



(a)



(b)

Fig. 5. Overall frequency response of the multiplexing networks where $f_0 = 19.9$ GHz. (a) Theoretical curves. (b) Experimental curves.

TABLE I

COMPONENTS OF MEASURED OVERALL LOSS

	MEASURED VALUE [a]	PREDICTED VALUE
OVERALL LOSS (dB)	5.0 dB	5.5 dB
V-H POLARIZING FILTER	0.16 X 2	0.20 X 2
TRANSMIT-RECIEVE FILTER	0.20 X 2	0.15 X 2
PRESSURE WINDOW	0.10 X 2	0.10 X 2
CHANNEL-DROPPING FILTER (FIVE CHANNELS IN CASCADE)	1.66	1.92
INTERCONNECTION WAVEGUIDES RECTANGULAR	0.40	0.60
FLEXIBLE	2.02	2.08

[a] When the center frequency is 19.9 GHz.

A large amount of flexibility for adjusting antenna direction and also for interconnecting to the repeater was included in the first time test. Therefore, the total length of the flexible waveguide extends to approximately 0.5 m for each transmit and receive side. In the future, it is promising that the overall loss will be reduced to within 4 dB of the first trial target because it will not be necessary for the flexible waveguide to be so long.

ACKNOWLEDGMENT

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A Potential Theory Method for Covered Microstrip

ANDREW FARRAR AND A. T. ADAMS

***Abstract*—Matrix methods [1] are used to analyze the properties of covered microstrip. The Green's function is calculated by a potential theory method assuming the TEM mode of propagation. Computed impedance values of covered microstrip agree closely with other experimental and theoretical data. The technique is a general one and can be used to treat multiple-layer and covered microstrip.**

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Introduction

The effect of a cover plate on the properties of microstrip transmission lines is of increasing importance because of high-power applications and emphasis on size reduction. The effect of the cover has been treated by Krage and Haddad [2], [3], by Priebe and Kyle [4] who use the method of Adams and Mautz [5], by Bryant and Weiss [6], and by Yamashita [7]. This short paper outlines a new method for treating microstrip, one that takes into account the presence of a cover and that can readily be extended to treat multiple-layered media.

Consider the covered microstrip transmission line shown in Fig. 1. First the Green's function, i.e., the potential function due to a line charge λ located at the interface parallel to the z axis Fig. 2, is obtained. Then the matrix methods [5] are used to compute the charge density and capacitance of the microstrip line of Fig. 1.

The potential V is given by the solution of Laplace's equation

$$\nabla^2 V = 0 \tag{1}$$

that has a solution of the form

$$V = (A \sin kx + B \cos kx)(C \sinh ky + D \cosh ky). \tag{2}$$

Specializing the solution to the problem in Fig. 2, one can write [8]

$$V_{\mathrm{I}} = \int_{-\infty}^{\infty} A(k) \cos kx \frac{\sinh ky}{\sinh hH} dk \tag{3}$$

$$V_{\mathrm{II}} = \int_{-\infty}^{\infty} A(k) \cos kx \frac{\sinh k(B-y)}{\sinh k(B-H)} dk \tag{4}$$

where k is the separation constant in Laplace's equation (1). k has a continuous range of allowed values since the geometry is unbounded in x.

The charge density on the plane $y = H$ (dielectric interface) is a function of x and the boundary condition to be satisfied is

$$\sigma_i = \lambda_i \delta(x) = D_{\mathrm{I}} - D_{\mathrm{II}} \big|_{y=H} \tag{5}$$

where λ_i is charge per unit length for the subsection i and D_{I} and D_{II} are the displacement vectors in regions I and II, respectively.

Substituting (3) and (4) into (5) gives the following expressions for the potentials in regions I and II

$$V_{\mathrm{I}} = \frac{\lambda_i}{2\pi} \int_{-\infty}^{\infty} \cdot \cos kx \frac{\sinh ky \sinh k(B-H)\, dk}{k[\epsilon_1 \cosh kH \sinh k(B-H) + \epsilon_2 \sinh kH \cosh k(B-H)]} \tag{6}$$

$$V_{\mathrm{II}} = \frac{\lambda_i}{2\pi} \int_{-\infty}^{\infty} \cdot \cos kx \frac{\sinh kH \sinh k(B-g)\, dk}{k[\epsilon_1 \cosh kH \sinh k(B-H) + \epsilon_2 \sinh ky \cosh k(B-H)]}. \tag{7}$$

Bryant and Weiss [9] evaluate integrals (3) and (4) by a computer algorithm. In this short paper an alternative method is used: the integrals (6) and (7) are evaluated by residue theory. There are an infinite number of poles, and an infinite series results. The locations of the poles are obtained by an iterative computer technique. The series is absolutely convergent. The potential thus evaluated represents the Green's function for the problem of Fig. 2.

Once the Green's function is evaluated, the potential problem of Fig. 1 may be treated by matrix methods. The strip is divided into N subsections and the following matrix equation is obtained

$$[V] = [D][\sigma] \tag{8}$$

where D_{ij} is the potential at subsection i due to unit charge density on subsection j obtained by specializing (6) or (7) to $y = H$. σ_j is the charge density of subsection j and V_i is the potential at subsection i. For convenience, subsection widths are assumed to be unity. The σ's are obtained by matrix inversion, the capacitance is obtained by summation, and the impedance is calculated by the relationship

$$Z_0 = \frac{1}{\nu_0 \sqrt{CC_0}} \tag{9}$$

where ν_0 is the velocity of light in vacuum, C_0 is the capacity per unit length for $\epsilon = \epsilon_0$, and C is the capacitance per unit length of the geometry of Fig. 1.

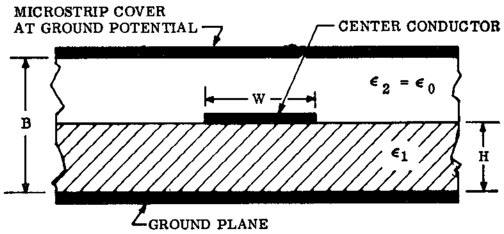

Fig. 1. Cross section of covered microstrip.

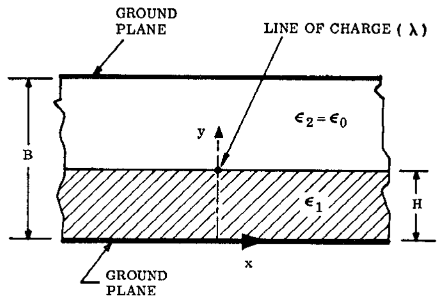

Fig. 2. Green's function geometry for covered microstrip.

The potential theory technique described here is a general one and it can readily be extended to calculate the properties of multiple microstrip lines with multiple layers of dielectric.

Computations

As an illustration of the present analysis, the characteristic impedance of the covered microstrip (see Fig. 2) was calculated for $\epsilon_{r1} = 9.6$ for different W/H ratios.

In this computation ratio, $B/H = \nu + 1$ was assumed to be an integer. Substituting this into (6) and (7), the Green's function for the problem becomes

$$D_{ij} = \frac{1}{\epsilon_0} \left\{ \frac{2}{\epsilon_r + \nu} \sum_{n=1}^{\infty} \frac{\exp\left[-\frac{n\pi |x_i - x_j|}{2H}\right]}{n\pi} + \sum_{m=1}^{\infty} \frac{\exp[-Z_m |x_i - x_j|]}{HZ_m[(\epsilon_r + \nu) - (1 + \epsilon\nu_r) \cot Z_m H \cot \nu Z_m H]} \right\}, \qquad x > 0 \tag{10}$$

where $\nu = 3, 5, 7, \cdots$ and Z_j are the poles of the integrand in (6), and x_i, x_j are coordinates of the field and source points, respectively. Since at $y = H$ the potential for $V_{\mathrm{I}} = V_{\mathrm{II}}$, there is no need carrying out the integration in (7). For $\nu = 2, 4, 6, \cdots$, the Green's function V_{I} is given by

$$D_{ij} = \frac{1}{\epsilon_0} \left\{ \sum_{m=1}^{\infty} \frac{\exp[-Z_m |x_i - x_j|]}{HZ_m[(\epsilon_r + \nu) - (1 + \epsilon_r \nu) \cot Z_m H \cot \nu Z_m H]} \right\} \tag{11}$$

and for $\nu = 1$

$$D_{ij} = \frac{2}{\epsilon_0(\epsilon_r + 1)} \sum_{n=1}^{\infty} \frac{\exp\left[-\frac{n\pi |x_i - x_j|}{2H}\right]}{n\pi}. \tag{12}$$

A computer program was prepared in Fortran IVand the properties of the covered microstrip line for $\epsilon_r = 9.6$ were calculated using (10). Fig. 3 shows the results. Notice that, as the ratio B/H increases, the impedance of the transmission line grows larger. As the microstrip cover is moved closer to the transmission line, the impedance decreases, as expected. The characteristic impedance of the unshielded microstrip ($B/H = \infty$), computed by matrix methods [11], is also plotted for comparison. For $\epsilon = \epsilon_0$, the data agree within ± 1

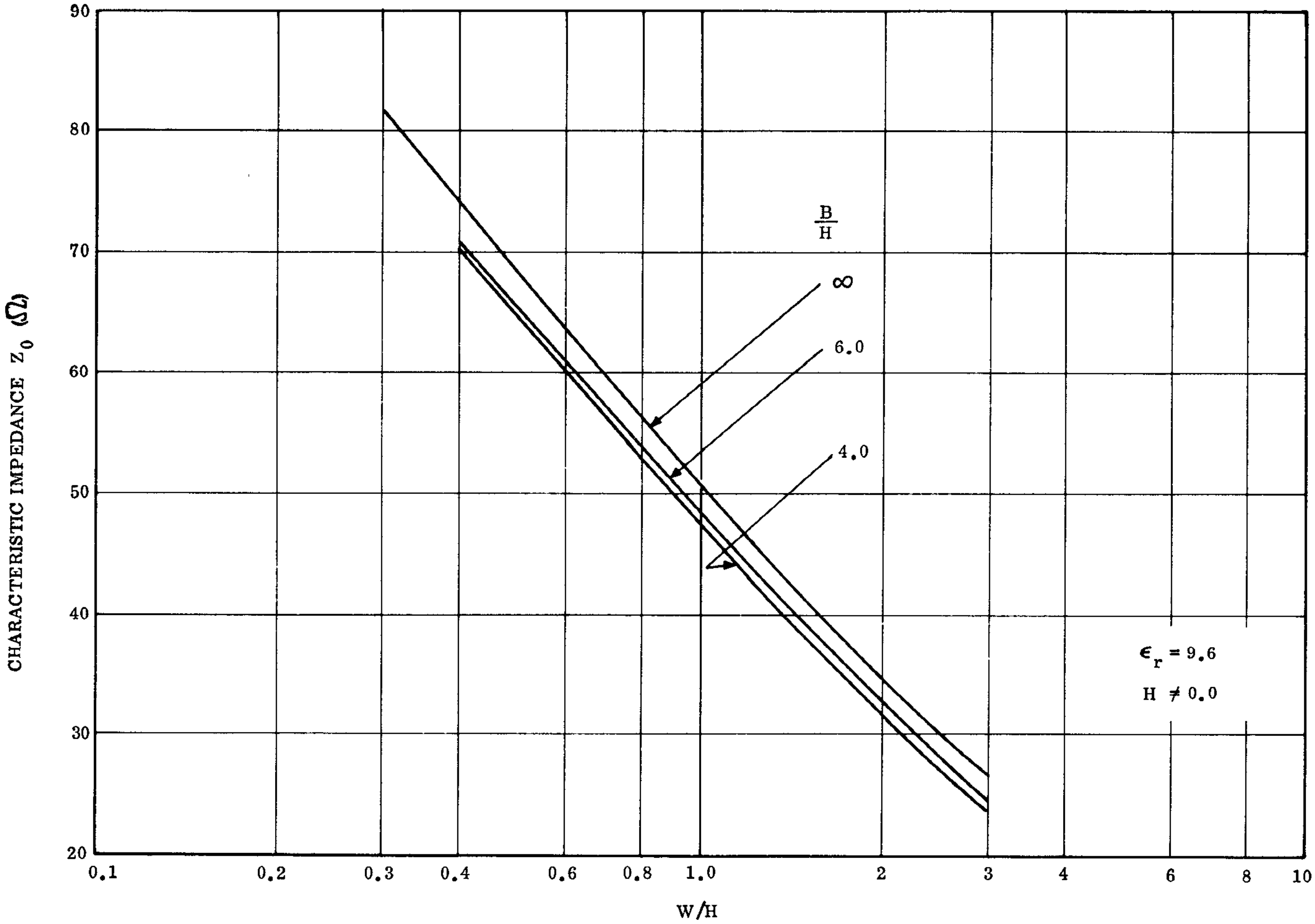

Fig. 3. Z_0 versus W/H for various values of B/H.

percent with that of Cohn [10]. For $\epsilon_r = 9.6$ the results were compared with that reported by Yamashita [7] who has carried out his calculations for sapphire ($\epsilon_r = 9.9$) using variational methods. Yamashita's results were higher by a maximum of 3 percent due to higher dielectric constant of sapphire. Using GE 635 computer, 3.6-s computation time is required to obtain one of the curves in Fig. 3.

Acknowledgment

The authors wish to thank C. E. Nelson of General Electric Company for helpful discussions.

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Spectral-Domain Approach for Calculating the Dispersion Characteristics of Microstrip Lines

TATSUO ITOH AND RAJ MITTRA

***Abstract*—The boundary value problem associated with the open microstrip line structure is formulated in terms of a rigorous, hybird-mode representation. The resulting equations are subsequently transformed, via the application of Galerkin's method in the spectral domain, to yield a characteristic equation for the dispersion properties of the open microstrip line.**

Numerical results are included for several different structural parameters. These are compared with other available data and with some experimental measurements.

Introduction

Because microwave integrated circuits are being used at higher frequencies, it is often necessary to predict the dispersion characteristics of microstrip lines and similar configurations. However, only very recently has the hybrid-mode analysis been applied for rigorous formulation of the dispersion problem for both the shileded [1]–[3] and open versions [4] of the microstrip lines.

The method followed by Denlinger [4] for analyzing the open microstrip line is critically dependent on the forms of the distribution one assumes, for the two current components on the center strip of the line, in the process of solving for the unknown amplitude of

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